Spiral plat avec une seule courbe terminale externe

Poids du spiral et anisochronisme en position verticale

Cas d'une montre bracelet

Caractéristiques du spiral *dextre*

Référence :C:\Résonateur (TA)\Data\Bal_spiral plat (ex num).mcd(R)

Référence :C:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $\quad ép = 0.03\,mm \quad ha = 0.15\,mm \quad S = 4.5\times 10^{-3}\,mm^2 \quad TOL := 10^{-12}$

$d2_{sp} = 4.52\,mm \quad d1_{sp} = 1.1\,mm \quad p_{sp} = 0.135\,mm \quad n_{sp} = 12.667$

$L := L_{sp} \quad L = 11.182\,cm \quad \psi_0 := 2\cdot\pi\cdot n_{sp} \quad \psi_0 = 4.56\times 10^3\,deg$

Position du point de raccordement sur le spiral $\quad \alpha_A := \pi \quad r_A := 0.5\cdot d2_{sp} \quad z_A := r_A\cdot e^{i\cdot\alpha_A}$

Forme initiale du spiral

$$a := \frac{p_{sp}}{2\cdot\pi} \qquad r_s(\alpha) := r_A - a\cdot(\alpha-\alpha_A) \qquad x_{0s}(\alpha) := r_s(\alpha)\cdot cos(\alpha) \qquad y_{0s}(\alpha) := r_s(\alpha)\cdot sin(\alpha)$$

$$s(\alpha) := \frac{1}{2\cdot a}\cdot\left(r_A{}^2 - r_s(\alpha)^2\right) \qquad s(\alpha) := r_A\cdot(\alpha-\alpha_A) - \frac{a}{2}\cdot(\alpha-\alpha_A)^2$$

Courbe terminale externe

$$r_{t1} := 0.8 \qquad r_{t1} := racine\left[(2\cdot r_{t1}-1)^4 - 4\cdot(1-r_{t1})^4 - \pi^2\cdot r_{t1}{}^2\cdot(1-r_{t1})^2, r_{t1}\right]\cdot r_A \qquad r_{t1} = 0.832\,r_A$$

$$r_{t2} := 2\cdot r_{t1} - r_A \qquad r_{t2} = 0.665\,r_A \qquad \beta_0 := arctan\left[\frac{\pi\cdot r_{t1}}{2\cdot(r_A - r_{t1})}\right] \qquad \beta_0 = 82.695\,deg \qquad l_t := r_{t2}\cdot\beta_0 + \pi\cdot r_{t1}$$

$$x_{0t1}(\alpha_t) := -r_A + r_{t1}\cdot(1+cos(\alpha_t)) \qquad y_{0t1}(\alpha_t) := r_{t1}\cdot sin(\alpha_t)$$

$$x_{0t2}(\beta_t) := r_{t2}\cdot cos(\beta_t) \qquad y_{0t2}(\beta_t) := r_{t2}\cdot sin(\beta_t)$$

Position des goupilles de raquettes $\quad r_{GR} := r_{t2} \quad \alpha_{GR} := -\beta_0 \quad \alpha_{GR} = -82.695\,deg$

$x_{GR} := x_{0t2}(\alpha_{GR}) \qquad y_{GR} := y_{0t2}(\alpha_{GR})$

Position du point d'attache à la virole $\quad r_V := 0.5\cdot d1_{sp} \quad \alpha_V(\theta) := \alpha_A + \psi_0 + \theta \quad r_B := r_V$

$x_V(\theta) := r_V\cdot cos(\alpha_V(\theta)) \qquad y_V(\theta) := r_V\cdot sin(\alpha_V(\theta)) \qquad L_t := L + l_t$

Amplitude stationnaire du balancier $\quad \theta_0 = 270\,deg$

Moment quadratique de section

Référence :C:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha)$

Calcul du déplacement de centre de gravité

$$z_{0s}(\alpha) := x_{0s}(\alpha) + i\cdot y_{0s}(\alpha) \qquad z_{0t1}(\alpha_t) := x_{0t1}(\alpha_t) + i\cdot y_{0t1}(\alpha_t) \qquad z_{0t2}(\beta_t) := x_{0t2}(\beta_t) + i\cdot y_{0t2}(\beta_t)$$

$$\sigma2 := \frac{1}{L_t}\cdot\left[\int_{\pi}^{\pi+\psi_0}\left(\left|z_{0s}(\alpha)\right|\right)^2\cdot r_s(\alpha)\,d\alpha + \int_0^{\pi}\left(\left|z_{0t1}(\alpha_t)\right|\right)^2\cdot r_{t1}\,d\alpha_t + \int_{-\beta_0}^{0}\left(\left|z_{0t2}(\beta_t)\right|\right)^2\cdot r_{t2}\,d\beta_t\right]$$

$$\sigma2 = 2.745\,mm^2$$

$s_s(\alpha) := s(\alpha) + l_t$ $\qquad$ $\kappa_s := \frac{1}{\sigma 2\cdot L_t^2}\cdot\int_{\pi}^{\pi+\psi_0} s_s(\alpha)\cdot\left(\left|z_{0s}(\alpha)\right|\right)^2\cdot r_s(\alpha)\, d\alpha$

$s_{t2}(\beta_t) := r_{t2}\cdot(\beta_0+\beta_t)$ $\qquad$ $s_{t1}(\alpha_t) := (r_{t2}\cdot\beta_0 + r_{t1}\cdot\alpha_t)$

$$\kappa_t := \frac{1}{\sigma 2\cdot L_t^2}\cdot\left[\int_{-\beta_0}^{0} s_{t2}(\beta_t)\cdot\left(\left|z_{0t2}(\beta_t)\right|\right)^2\cdot r_{t2}\, d\beta_t + \int_0^{\pi} s_{t1}(\alpha_t)\cdot\left(\left|z_{0t1}(\alpha_t)\right|\right)^2\cdot r_{t1}\, d\alpha_t\right]$$

$\kappa := \kappa_t + \kappa_s$

$\kappa = 0.367$

$$\mathbf{\Delta_s}(\theta) := \frac{i\cdot\theta}{L_t}\cdot\int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha)\cdot exp\left(i\cdot\theta\cdot\frac{s_s(\alpha)}{L_t}\right)\cdot r_s(\alpha)\, d\alpha$$

$\mathbf{\Delta_s}(\theta_0) = 0.186 + 0.048i\, mm$

$$\mathbf{\Delta_t}(\theta) := \frac{i\cdot\theta}{L_t}\cdot\left(\int_{-\beta_0}^{0} z_{0t2}(\beta_t)\cdot exp\left(i\cdot\frac{\theta}{L_t}\cdot s_{t2}(\beta_t)\right)\cdot r_{t2}\, d\beta_t + \int_0^{\pi} z_{0t1}(\alpha_t)\cdot exp\left(i\cdot\frac{\theta}{L_t}\cdot s_{t1}(\alpha_t)\right)\cdot r_{t1}\, d\alpha_t\right)$$

$\mathbf{\Delta_t}(\theta_0) = -0.162 - 0.047i\, mm$

$\mathbf{\Delta_1}(\theta) := \mathbf{\Delta_t}(\theta) + \mathbf{\Delta_s}(\theta)$ $\qquad$ $\mathbf{\Delta_1}(\theta_0) = 0.024 + 7.91i\times 10^{-4}\, mm$

$\zeta(\theta) := -i\cdot\frac{d}{d\theta}\mathbf{\Delta_1}(\theta) - \kappa\cdot\mathbf{\Delta_1}(\theta)$ $\qquad$ $\zeta(\theta_0) = 3.409\times 10^{-3} - 0.014i\, mm$

$$\zeta_s(\theta) := \frac{1}{L_t}\cdot\int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha)\cdot e^{i\cdot\theta\cdot\frac{s_s(\alpha)}{L_t}}\cdot\left[1 + i\cdot\theta\cdot\left(\frac{s_s(\alpha)}{L_t} - \kappa\right)\right]\cdot r_s(\alpha)\, d\alpha$$

$$\zeta_{t2}(\theta) := \frac{1}{L_t}\cdot\int_{-\beta_0}^{0} z_{0t2}(\beta_t)\cdot e^{i\cdot\theta\cdot\frac{s_{t2}(\beta_t)}{L_t}}\cdot\left[1 + i\cdot\theta\cdot\left(\frac{s_{t2}(\beta_t)}{L_t} - \kappa\right)\right]\cdot r_{t2}\, d\beta_t$$

$$\zeta_{t1}(\theta) := \frac{1}{L_t}\cdot\int_{0}^{\pi} z_{0t1}(\alpha_t)\cdot e^{i\cdot\theta\cdot\frac{s_{t1}(\alpha_t)}{L_t}}\cdot\left[1 + i\cdot\theta\cdot\left(\frac{s_{t1}(\alpha_t)}{L_t} - \kappa\right)\right]\cdot r_{t1}\, d\alpha_t$$

$\zeta_t(\theta) := \zeta_{t2}(\theta) + \zeta_{t1}(\theta)$

$\zeta(\theta) := \zeta_t(\theta) + \zeta_s(\theta)$ $\qquad$ $\zeta(\theta_0) = 3.409\times 10^{-3} - 0.014i\, mm$

Approximations de Haag

Paramètres de la courbe terminale externe

$X_{0t1}(\alpha_t) := r_A - r_{t1} + r_{t1}\cdot cos(\alpha_t)$ $\qquad$ $Y_{0t1}(\alpha_t) := r_{t1}\cdot sin(\alpha_t)$ $\qquad$ $X_{0t2}(\beta_t) := -r_{t2}\cdot cos(\beta_t)$ $\qquad$ $Y_{0t2}(\beta_t) := -r_{t2}\cdot sin(\beta_t)$

$$X_1 := \frac{1}{r_A^2}\cdot\left(\int_0^{\pi} X_{0t1}(\alpha)\cdot r_{t1}\, d\alpha + \int_0^{\beta_0} X_{0t2}(\beta)\cdot r_{t2}\, d\beta\right)$$

$X_1 = 0$

$$Y_1 := \frac{1}{r_A^2}\cdot\left(\int_0^{\pi} Y_{0t1}(\alpha)\cdot r_{t1}\, d\alpha + \int_0^{\beta_0} Y_{0t2}(\beta)\cdot r_{t2}\, d\beta\right) - 1$$

$Y_1 = 0$

$\rho_1 := \sqrt{X_1^{\,2} + Y_1^{\,2}}$ $\qquad \varphi_1 := Atan(X_1, Y_1)$ $\qquad \rho_1 = 0$ $\qquad \varphi_1 = 270\,deg$

$$X_2 := \frac{1}{r_A^{\,3}} \cdot \left[\int_0^{\pi} r_{t1} \cdot \alpha \cdot X_{0t1}(\alpha) \cdot r_{t1}\, d\alpha + \int_0^{\beta_0} (r_{t1} \cdot \pi + r_{t2} \cdot \beta) \cdot X_{0t2}(\beta) \cdot r_{t2}\, d\beta \right] + 1$$

$$Y_2 := \frac{1}{r_A^{\,3}} \cdot \left[\int_0^{\pi} r_{t1} \cdot \alpha \cdot Y_{0t1}(\alpha) \cdot r_{t1}\, d\alpha + \int_0^{\beta_0} (r_{t1} \cdot \pi + r_{t2} \cdot \beta) \cdot Y_{0t2}(\beta) \cdot r_{t2}\, d\beta \right]$$

$\rho_2 := \sqrt{X_2^{\,2} + Y_2^{\,2}}$ $\qquad \varphi_2 := Atan(X_2, Y_2)$ $\qquad \rho_2 = 1.055$ $\qquad \varphi_2 = 147.579\,deg$

Formule de Haag avec κ = 1/3 $\qquad \boldsymbol{OA} := r_A \cdot e^{i \cdot \pi}$ $\qquad \boldsymbol{OB} := r_B \cdot e^{i \cdot (\pi + \psi_0)}$

$$\zeta_{1ah}(\rho_1, \theta) := \frac{1}{3 \cdot L_t} \cdot \left[r_A \cdot \rho_1 \cdot e^{-i \cdot \varphi_1} \cdot (3 - i \cdot \theta) - \frac{\theta}{L_t} \cdot r_A^{\,2} \cdot \rho_2 \cdot e^{-i \cdot \varphi_2} \cdot (6 \cdot i + \theta) \right] \cdot \boldsymbol{OA}$$

$$\zeta_{2ah}(\theta) := \frac{1}{3 \cdot L_t} \cdot \left[-(i \cdot r_B + 2 \cdot a) \cdot (3 + 2 \cdot i \cdot \theta) + \frac{\theta}{L_t} \cdot r_B^{\,2} \cdot (6 \cdot i - 2 \cdot \theta) \right] \cdot \boldsymbol{OB} \cdot e^{i \cdot \theta}$$

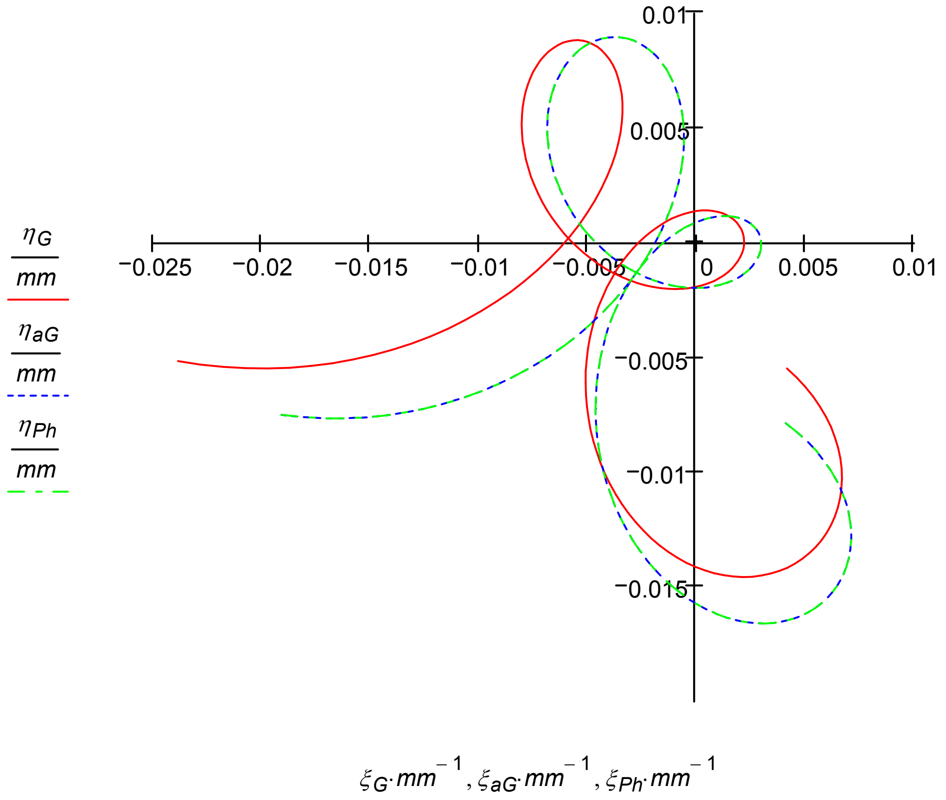
$$\zeta_{ah}(\rho_1, \theta) := \zeta_{1ah}(\rho_1, \theta) + \zeta_{2ah}(\theta)$$

Courbes Phillips $\qquad \zeta_{aPh}(\theta) := \zeta_{ah}(0, \theta)$

Graphes du déplacement du centre de gravité

$n := 201$ $\qquad i := 0 .. n - 1$ $\qquad \Delta\theta := \dfrac{4 \cdot \pi}{n - 1}$ $\qquad \theta_i := -2 \cdot \pi + i \cdot \Delta\theta$ $\qquad \xi_{G_i} := Re(\zeta(\theta_i))$ $\qquad \eta_{G_i} := Im(\zeta(\theta_i))$

$\xi_{aG_i} := Re(\zeta_{ah}(\rho_1, \theta_i))$ $\qquad \eta_{aG_i} := Im(\zeta_{ah}(\rho_1, \theta_i))$ $\qquad \xi_{Ph_i} := Re(\zeta_{aPh}(\theta_i))$ $\qquad \eta_{Ph_i} := Im(\zeta_{aPh}(\theta_i))$



Perturbation de période - spiral non déformé en position de repos

Calcul par intégrations numériques

$\eta(\theta) := Im(\zeta(\theta))$ $\quad$ $Gamma(\theta) := -m_s \cdot g \cdot \frac{d}{d\theta}\eta(\theta)$

$\theta(\varphi) := \theta_0 \cdot cos(\varphi)$ $\quad$ $Delta(\theta_0) := \frac{L}{2\cdot\pi\cdot\theta_0\cdot E\cdot I_{33}} \cdot \int_0^{2\cdot\pi} Gamma(\theta_0\cdot cos(\varphi))\cdot cos(\varphi$ $\quad$ $Delta(\theta_0) = 2.584 \times 10^{-5}$

$$Z_s(\theta_0) := \frac{1}{L_t^2}\cdot\int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha)\cdot s_s(\alpha)\cdot\left[\left(\kappa - \frac{s_s(\alpha)}{L_t}\right)\cdot J0\left(\theta_0\cdot\frac{s_s(\alpha)}{L_t}\right) - \frac{1}{\theta_0}\cdot J1\left(\theta_0\cdot\frac{s_s(\alpha)}{L_t}\right)\right]\cdot r_s(\alpha)\, d\alpha$$

$$Z_{t2}(\theta_0) := \frac{r_{t2}}{L_t^2}\cdot\int_{-\beta_0}^{0} z_{0t2}(\beta_t)\cdot s_{t2}(\beta_t)\cdot\left[\left(\kappa - \frac{s_{t2}(\beta_t)}{L_t}\right)\cdot J0\left(\theta_0\cdot\frac{s_{t2}(\beta_t)}{L_t}\right) - \frac{1}{\theta_0}\cdot J1\left(\theta_0\cdot\frac{s_{t2}(\beta_t)}{L_t}\right)\right] d\beta_t$$

$$Z_{t1}(\theta_0) := \frac{r_{t1}}{L_t^2}\cdot\int_{0}^{\pi} z_{0t1}(\alpha_t)\cdot s_{t1}(\alpha_t)\cdot\left[\left(\kappa - \frac{s_{t1}(\alpha_t)}{L_t}\right)\cdot J0\left(\theta_0\cdot\frac{s_{t1}(\alpha_t)}{L_t}\right) - \frac{1}{\theta_0}\cdot J1\left(\theta_0\cdot\frac{s_{t1}(\alpha_t)}{L_t}\right)\right] d\alpha_t$$

$Z(\theta_0) := Z_{t2}(\theta_0) + Z_{t1}(\theta_0) + Z_s(\theta_0)$ $\quad$ $Z(\theta_0) = 1.982 \times 10^{-4} - 4.213i \times 10^{-4}\, mm$

$Delta(\theta_0) := -g\cdot\frac{m_s\cdot L}{E\cdot I_{33}}\cdot Im(Z(\theta_0))$ $\quad$ $Delta(\theta_0) = 2.584 \times 10^{-5}$

$\mu(\theta_0) := -86400\cdot Delta(\theta_0)$ $\quad$ $\boxed{\mu(\theta_0) = -2.232}$ $\quad$ $\boxed{\mu(180\cdot deg) = -1.219}$

Approximations de Haag

$f(\theta_0, s) := \frac{s}{L_t}\cdot\left[\left(\kappa - \frac{s}{L_t}\right)\cdot J0\left(\theta_0\cdot\frac{s}{L_t}\right) - \frac{1}{\theta_0}\cdot J1\left(\theta_0\cdot\frac{s}{L_t}\right)\right]$ $\quad$ $f_1(\theta_0, s) := \frac{d}{ds}f(\theta_0, s)$

$$Z_a(\theta_0) := \frac{1}{L_t}\cdot\left[\left(r_A\cdot\rho_1\cdot e^{-i\cdot\varphi_1} + 2\cdot a\right)\cdot f(\theta_0, l_t) - r_A^2\cdot\rho_2\cdot e^{-i\cdot\varphi_2}\cdot f_1(\theta_0, l_t)\right]\cdot \mathbf{OA}$$

$$Z_a(\theta_0) := Z_a(\theta_0) + \frac{1}{L_t}\cdot\left[-(i\cdot r_B + 2\cdot a)\cdot f(\theta_0, l_t + L) + r_B^2\cdot f_1(\theta_0, l_t + L)\right]\cdot \mathbf{OB}$$

$\delta_a(\theta_0) := -g\cdot\frac{m_s\cdot L}{E\cdot I_{33}}\cdot Im(Z_a(\theta_0))$ $\quad$ $\delta_a(\theta_0) = 2.418 \times 10^{-5}$

$\mu_a(\theta_0) := -86400\cdot\delta_a(\theta_0)$ $\quad$ $\boxed{\mu_a(\theta_0) = -2.089}$ $\quad$ $\boxed{\mu_a(180\cdot deg) = -1.078}$

$F(\theta_0) := J0(\theta_0) - \theta_0\cdot J1(\theta_0)$ $\quad$ $F_1(\theta_0) := (1-\kappa)\cdot J0(\theta_0) + \frac{1}{\theta_0}\cdot J1(\theta_0)$ $\quad$ $F_2(\theta_0) := 2\cdot J0(\theta_0) + (1-\kappa)\cdot F(\theta_0)$

$$Z_{ah}(\theta_0) := \frac{\kappa}{L_t^2}\cdot\left[\left(r_A\cdot\rho_1\cdot e^{-i\cdot\varphi_1} + 2\cdot a\right)\cdot l_t - r_A^2\cdot\rho_2\cdot e^{-i\cdot\varphi_2}\right]\cdot \mathbf{OA}$$

$$Z_{ah}(\theta_0) := Z_{ah}(\theta_0) - \frac{1}{L_t}\left[-(i\cdot r_B + 2\cdot a)\cdot F_1(\theta_0) + \frac{r_B^2}{L_t}\cdot F_2(\theta_0)\right]\cdot \mathbf{OB}$$

$$\delta_{ah}(\theta_0) := -g\cdot\frac{m_s\cdot L}{E\cdot I_{33}}\cdot Im(Z_{ah}(\theta_0)) \qquad \delta_{ah}(\theta_0) = 3.034\times 10^{-5}$$

$$\mu_{ah}(\theta_0) := -86400\cdot\delta_{ah}(\theta_0) \qquad \boxed{\mu_{ah}(\theta_0) = -2.621} \qquad \boxed{\mu_{ah}(180\cdot deg) = -1.585}$$

Cas de courbes Phillips

$$Z_{Ph}(\theta_0) := \frac{\kappa}{L_t^{\,2}}\cdot\left(2\cdot a\cdot l_t - r_A^{\,2}\cdot\rho_2\cdot e^{-i\cdot\varphi_2}\right)\cdot\mathbf{OA} - \frac{1}{L_t}\cdot\left[-(i\cdot r_B + 2\cdot a)\cdot F_1(\theta_0) + \frac{r_B^{\,2}}{L_t}\cdot F_2(\theta_0)\right]\cdot\mathbf{OB}$$

$$\delta_{Ph}(\theta_0) := -g\cdot\frac{m_s\cdot L}{E\cdot I_{33}}\cdot Im(Z_{Ph}(\theta_0)) \qquad \delta_{Ph}(\theta_0) = 3.034\times 10^{-5}$$

$$\mu_{Ph}(\theta_0) := -86400\cdot\delta_{Ph}(\theta_0) \qquad \boxed{\mu_{Ph}(\theta_0) = -2.621} \qquad \boxed{\mu_{Ph}(180\cdot deg) = -1.585}$$

Approximations supplémentaires avec κ = 1/3 et $l_t << L_t$

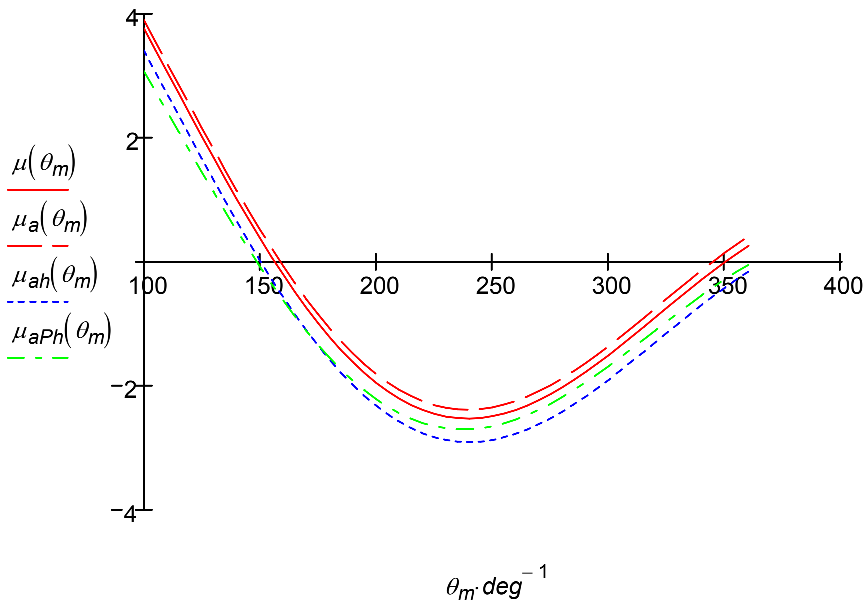
$$U(\theta_0) := \frac{2}{3}\cdot J0(\theta_0) + \frac{1}{\theta_0}\cdot J1(\theta_0)$$

$$Z_{aPh}(\theta_0) := \frac{-r_A^{\,2}}{3\cdot L_t^{\,2}}\cdot\rho_2\cdot e^{-i\cdot\varphi_2}\cdot\mathbf{OA} + \frac{i\cdot r_B}{L_t}\cdot U(\theta_0)\cdot\mathbf{OB}$$

$$\delta_{aPh}(\theta_0) := -g\cdot\frac{m_s\cdot L}{E\cdot I_{33}}\cdot Im(Z_{aPh}(\theta_0)) \qquad \delta_{aPh}(\theta_0) = 2.762\times 10^{-5}$$

$$\mu_{aPh}(\theta_0) := -86400\cdot\delta_{aPh}(\theta_0) \qquad \boxed{\mu_{aPh}(\theta_0) = -2.386} \qquad \boxed{\mu_{aPh}(180\cdot deg) = -1.552}$$

$$\theta_m := 100\cdot deg, 105\cdot deg .. 360\cdot deg$$



Influence de la position du point d'attache

$\boldsymbol{OB}(\alpha_B) := r_B \cdot e^{i \cdot \alpha_B}$ $\qquad \alpha_{B1} := 0 \qquad \alpha_{B2} := \pi \qquad \varphi_2(k, p) := 2 \cdot k \cdot \pi + p \cdot \frac{\pi}{2} - \psi_0$

$$Z_{aPh}(\theta_0, \alpha_B, p) := \left[\frac{-r_A^{\,2}}{3 \cdot L_t^{\,2}} \cdot \rho_2 \cdot e^{-i \cdot \left(p \cdot \frac{\pi}{2}\right)} \cdot \frac{r_A}{r_B} + \frac{i \cdot r_B}{L_t} \cdot U(\theta_0) \right] \cdot \boldsymbol{OB}(\alpha_B)$$

$\delta_{aPh}(\theta_0, \alpha_B, p) := -g \cdot \frac{m_s \cdot L}{E \cdot I_{33}} \cdot Im(Z_{aPh}(\theta_0, \alpha_B, p))$ $\qquad \mu_{aPh}(\theta_0, \alpha_B, p) := -86400 \cdot \delta_{aPh}(\theta_0, \alpha_B, p)$